

CODING TECHNIQUES FOR ANALOG SOURCES

Prof.Pratik Tawde

*Lecturer, Electronics and Telecommunication Department,
Vidyalankar Polytechnic, Wadala (India)*

ABSTRACT

Image Compression is a process of removing redundant pixels from an image. There are various Image Compression Techniques available. Predictive Coding is one of the basic Image Compression Techniques. In Predictive Coding Pulse-code modulation (PCM) is a basic technique for image compression. In case of PCM the rate of the bit stream is simply reduced by removing a fixed number of least significant bits from each codeword so PCM coding technique is extremely simple but it has a poor coding efficiency. Another Predictive Coding technique is known as the differential pulse code modulation (DPCM).

Keywords: Predictive Coding, JPEG, DPCM and Complexity

I. INTRODUCTION

Images and videos are moved around the World Wide Web by millions of users almost in a nonstop fashion, and then, there is television (TV) transmission round the clock. This process of reducing the image and video data so that it fits into the available limited bandwidth or storage space is termed data compression. Data compression refers to the process of reducing the digital source data to a desired level and bandwidth compression refers to the process of reducing the analog bandwidth of the analog source. Today, most signals of interest (e.g., voice, audio, image, video) are digitally acquired (digitized) using A/D converters. A/D converters perform pulse-code modulation (PCM) with uniform quantization and fixed-length binary coding.

1. Temporal waveform coding 2.Spectral waveform coding 3.Model-based coding

Temporal Waveform Coding- In this type of encoding, the source encoder is designed to represent digitally the temporal characteristics of the source waveform.

Spectral Waveform Coding- The signal waveform is usually subdivided into different frequency bands, and either the time waveform in each band or its spectral characteristics are encoded for transmission.

Model-based coding- It is based on a mathematical model of the source.

II. OPTIMUM QUANTIZATION

Quantization of the amplitudes of the sampled signal results in data compression, but it also introduces some distortion of the waveform or a loss of signal fidelity.

2.1 Rate-Distortion Function R(D)

The minimum rate in bits per source output that is required to represent the output X of the memoryless source with a distortion less than or equal to D is called the rate-distortion function R(D).

Distortion of the general form:

$$d(x_k, \tilde{x}_k) = |x - \tilde{x}_k|^p$$

The distortion between a sequence of samples X_n and the corresponding quantized values \tilde{X}_n

$$D = E[d(X_n, \tilde{X}_n)] = \frac{1}{n} \sum_{k=1}^n E[d(x_k, \tilde{x}_k)]$$

$$R(D) = \min_{p(\tilde{x}|x): E[d(X, \tilde{X})] \leq D} I(X; \tilde{X})$$

$I(X; \tilde{X})$ is the average mutual information between X and \tilde{X} . Note that $R(D)$ decreases as D increases.

2.2 Theorem: Rate-Distortion Function for a Memoryless Gaussian Source

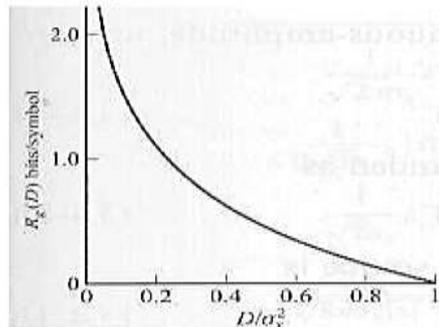
The minimum information rate necessary to represent the output of a discrete-time, continuous-amplitude memoryless Gaussian source based on a mean-square-error distortion measure per symbol (single letter distortion measure) is:

$$R_x(D) = \begin{cases} \frac{1}{2} \log_2 \left(\frac{\sigma_x^2}{D} \right) & (0 \leq D \leq \sigma_x^2) \\ 0 & (D > \sigma_x^2) \end{cases}$$

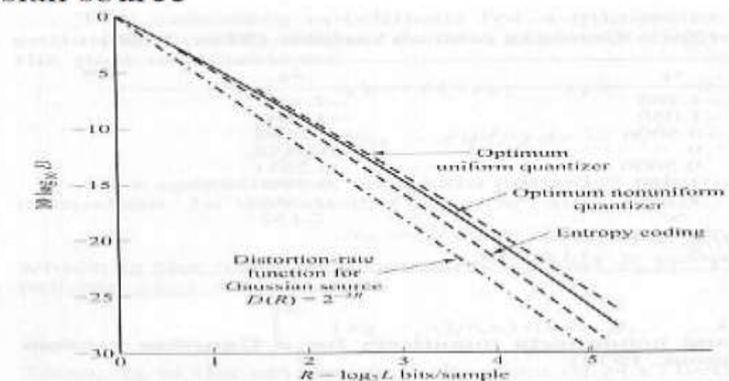
σ_x^2 is the variance of the Gaussian source output.

Rate-distortion function for a memoryless Gaussian source

- No information need be transmitted when the distortion $D \geq \sigma_x^2$.
- $D = \sigma_x^2$ can be obtained by using zeros in the reconstruction of the signal.
- For $D > \sigma_x^2$ we can use statistically independent, zero-mean Gaussian noise samples with a variance of $D - \sigma_x^2$ for the reconstruction.



Distortion versus rate curves for discrete-time memoryless Gaussian source



2.3 Temporal Waveform Coding

Time Domain Characteristics of signal can be represented by following popular methods.

1. Pulse Code Modulation (PCM)

2. Differential Pulse Code Modulation (DPCM)
3. Delta Modulation (DM)

2.4 Pulse Code Modulation (PCM)

A schematic diagram for Pulse Code Modulation is shown in Fig. 1

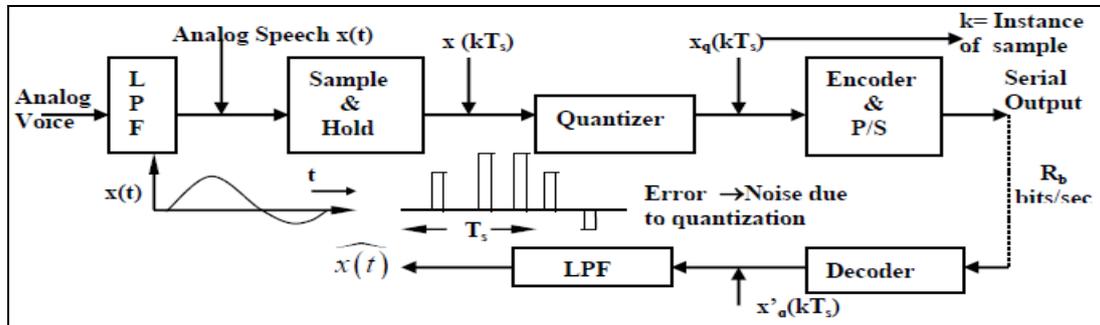


Fig.1 Schematic diagram of a PCM coder – decoder

- The signal is band limited by the low pass filter.
- Let $X(t)$ denote the filtered signal to be coded. The process of analog to digital conversion primarily involves three operations:
 - (a) Sampling of $X(t)$,
 - (b) Quantization (i.e. approximation) of the discrete time samples, $X(kT_s)$ and
 - (c) Suitable encoding of the quantized time samples $X_q(kT_s)$. T_s indicates the sampling interval where $R_s = 1/T_s$ is the sampling rate (samples /sec). A standard sampling rate for speech signal, band limited to 3.4 kHz, is 8 Kilo-samples per second ($T_s = 125\mu$ sec), thus, obeying Nyquist’s sampling theorem.

2.5 Quantization

Quantization is an approximation process and thus, causes some distortion in the reconstructed analog signal. We say that quantization contributes to “noise”.

- Below are Input / Output characteristics of Quantizer. The input signal range ($\pm V$) of the quantizer has been divided in eight equal intervals. The width of each interval, δ , is known as the step size. While the amplitude of a time sample $x(kT_s)$ may be any real number between $+V$ and $-V$, the quantizer presents only one of the allowed eight values ($\pm\delta/2, \pm3\delta/2, \dots$) depending on the proximity of $x(kT_s)$ to these levels.

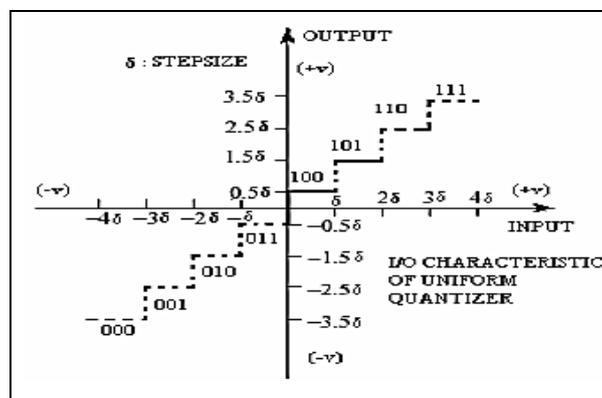


Fig 2 Input / Output Characteristics of Quantizer

- The quantizer of Fig 2 is known as “mid-riser” type. For such a mid-riser quantizer, a slightly positive and a slightly negative values of the input signal will have different levels at output. This may be a problem when the speech signal is not present but small noise is present at the input of the quantizer.
- To avoid such a random fluctuation at the output of the quantizer, the “mid-tread” type uniform quantizer Fig 3 may be used.

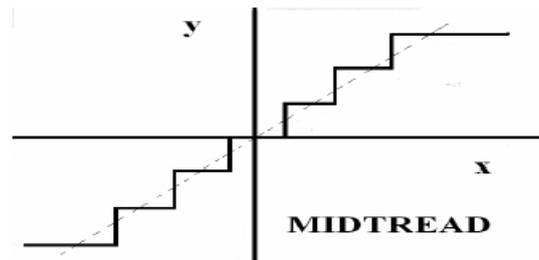


Fig 3 Mid-Tread Type Uniform Quantizer Characteristics

2.6 Encoding

Encoding is used to translate the Discrete set of sample values to more appropriate signal called Code. Suppose in binary code word ‘n’ bits are used, then we may represent 2^n . After coding binary signal is represented by train of pulses as NRZ , RZ unipolar or bipolar.

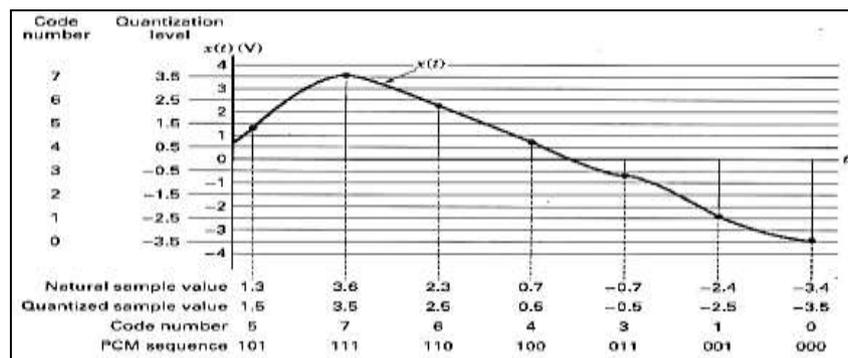
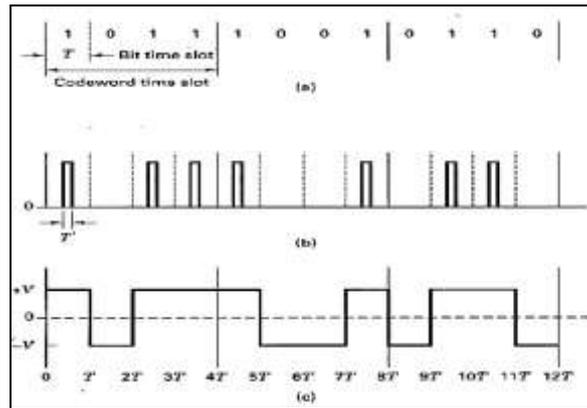


Fig 4 Natural Samples, Quantized Samples, and Pulse Code Modulation

- The PCM coded bit stream may be taken for further digital signal processing and modulation for the purpose of transmission.
- The PCM decoder at the receiver expects a serial or parallel bit-stream at its input so that it can decode the respective groups of bits (as per the encoding operation) to generate quantized sample sequence $[x'_q(kTs)]$.
- Following Nyquist’s sampling theorem for band limited signals, the low pass reconstruction filter whose $f_c = \text{message BW}$ produces a close replica $\hat{x}(t)$ of the original speech signal $x(t)$.



**Fig 5 (a) PCM Sequence. (b) Pulse Representation of PCM.
(c) Pulse waveform (transition between two levels).**

2.7 Multiplexing

- Different message sources are Time – Multiplexed for this receiver & transmitter are synchronized.

2.8 Channel Noise & Error Probability

The Performance of PCM system is influenced by two major sources of Noise.

1. Channel Noise: Introduced in transmission path
2. Quantizing Noise: Introduced in transmitter

2.9 Channel Noise

Due to Channel Noise Symbol '0' appears as '1' & Vice versa.

Probability of error $P_e = 1/2 * \text{erfc} (1/2 * (E_{\text{max}} / N_o)^{1/2})$, Where N_o is noise power.

2.10 Quantizing Noise

Is produced at transmitter of PCM by rounding off analog sample value to nearby permissible level. Quantizing

Noise $\sigma_Q^2 = \Delta^2 / 12$, Where Δ is step size

2.11 Characteristics of PCM

- Average Probability of error depends on ratio of Peak Signal energy to Noise spectral energy.
- In PCM signal is regenerated so effects of amplitude, phase & nonlinear effects in one link has no effect on next link.
- Transmission requirement PCM link are independent of total length of system.
- PCM is very rugged system, means less noise effect unless noise amplitude is greater than half of pulse height.

Advantages: In PCM signal is regenerated so effects of amplitude, phase & nonlinear effects in one link has no effect on next link. Transmission requirement PCM link are independent of total length of system.

Disadvantages: High bit rate & noise limits the use.

III. DPCM

- In PCM Samples of signal are usually correlated as amplitude of signal does not change much ie signal is correlated or carries redundant information. This aspect of speech signal is exploited in differential pulse code modulation (DPCM) technique.

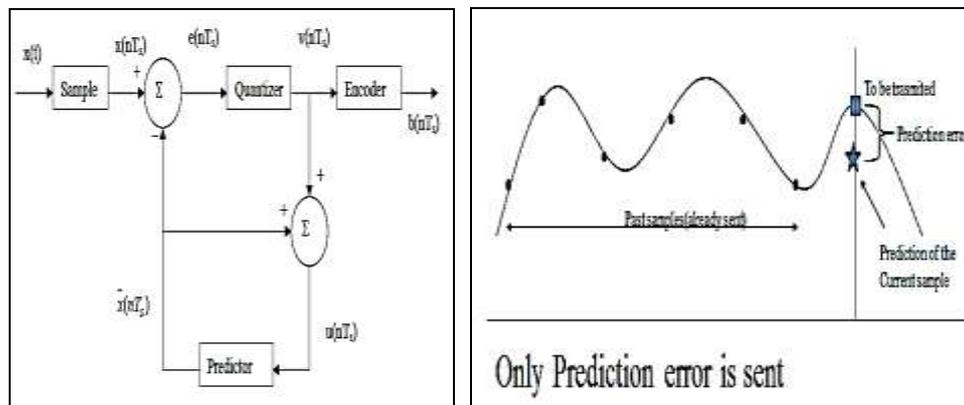


Fig.6 Schematic Diagram of a DPCM Modulator

- A schematic diagram for the basic DPCM modulator is shown in Fig 6 Note that a predictor block, a summing unit and a subtraction unit have been strategically added to the chain of blocks of PCM coder instead of feeding the sampler output $x(kT_s)$ directly to a linear quantizer. An error sample $e_p(kT_s)$ is fed.
- The error sample is given by the following expression:

$$e_p(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$\hat{x}(nT_s)$ is a predicted value for $x(nT_s)$ and is supposed to be close to $x(nT_s)$ such that $e_p(nT_s)$ is very small in magnitude $e_p(nT_s)$ is called as the 'prediction error for the n^{th} sample'.

- We envisage smaller step size for the linear quantizer compared to the step size of an equivalent PCM quantizer. As a result, it should be possible to achieve higher SQNR for DPCM codec delivering bits at the same rate as that of a PCM codec. There is another possibility of decreasing the coded bit rate compared to a PCM system if an SQNR as achievable by a PCM codec with linear equalizer is sufficient.
- A block schematic diagram of a DPCM demodulator is shown in Fig 7. The scheme is straightforward and it tries to estimate $u(kT_s)$ using a predictor unit identical to the one used in the modulator. We have already observed that $u(kT_s)$ is very close to $x(kT_s)$ within a small quantization error of $q(kT_s)$. The analog speech signal is obtained by passing the $\hat{u}(kT_s)$ through an appropriate low pass filter.

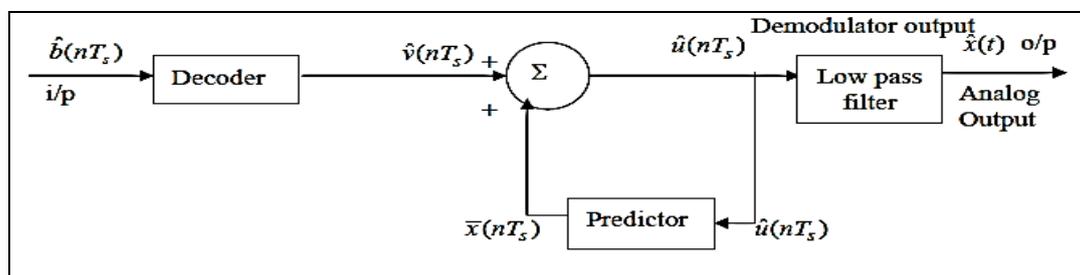


Fig 7 Schematic Diagram of a DPCM Demodulator

Advantages: Less bit rate generated so better utilization of bandwidth. Redundant information is less carried

Disadvantages: Predictor increase hardware complexity of system.

Delta Modulation (DM)

- If the sampling interval 'T_s' in DPCM is reduced considerably, i.e. if we sample a band limited signal at a rate much faster than the Nyquist sampling rate, the adjacent samples should have higher correlation. The sample-to-sample amplitude difference will usually be very small. So, one may even think of only 1-bit quantization of the difference signal. The principle of Delta Modulation (DM) is based on this premise.

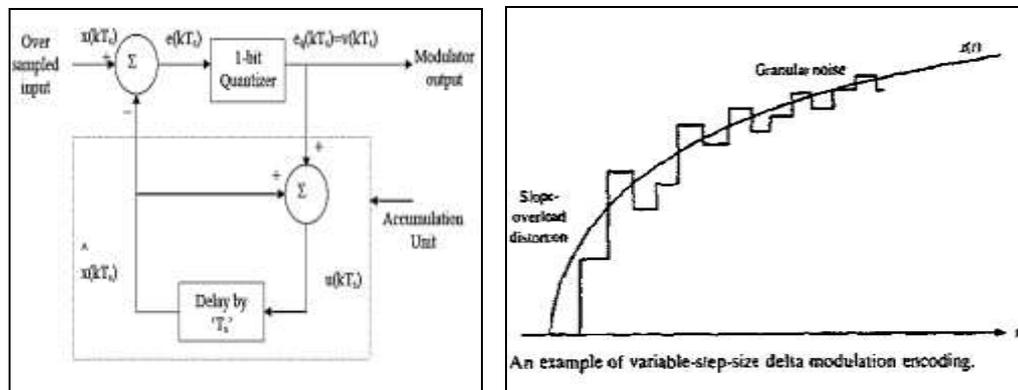


Fig. 8 Block Diagram of a Delta Modulator

- Delta modulation is also viewed as a 1-bit DPCM scheme. The 1-bit quantizer is equivalent to a two-level comparator (also called as a hard limiter). Fig.8 shows the schematic arrangement for generating a delta-modulated signal.
- Note that,

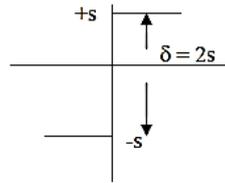
$$\begin{aligned} e(kT_s) &= x(kT_s) - \hat{x}(kT_s) \\ &= x(kT_s) - u([k-1]T_s) \end{aligned}$$

3.1 Features of Delta Modulation

- No effective prediction unit – the prediction unit of a DPCM coder (Fig. 8) is eliminated and replaced by a single-unit delay element.
- A 1-bit quantizer with two levels is used. The quantizer output simply indicates whether the present input sample $x(kT_s)$ is more or less compared to its accumulated approximation $\hat{x}(kT_s)$
- Output $\hat{x}(kT_s)$ of the delay unit changes in small steps.
- The accumulator unit goes on adding the quantizer output with the previous accumulated version $\hat{x}(kT_s)$. .
- $u(kT_s)$, is an approximate version of $x(kT_s)$.
- Performance of the Delta Modulation scheme is dependent on the sampling rate.
- Most of the above comments are acceptable only when two consecutive inputsamples are very close to each other.

$$\begin{aligned} e(kT_s) &= x(kT_s) - \{\hat{x}([k-1]T_s) + v([k-1]T_s)\} \\ \text{Further,} \quad v(kT_s) &= e_q(kT_s) = s \cdot \text{sign}[e(kT_s)] \end{aligned}$$

Here, 's' is half of the step-size δ as indicated in Fig 9 below



This diagram indicates the output levels of 1-bit quantizer. Note that if δ is the step size, the two output levels are $\pm s$

Now, assuming zero initial condition of the accumulator, it is easy to see

$$u(kT_s) = s \cdot \sum_{j=1}^k \text{sign}[e(jT_s)]$$

$$u(kT_s) = \sum_{j=1}^k v(jT_s)$$

Further,

$$\hat{x}(kT_s) = u([k-1]T_s) = \sum_{j=1}^{k-1} v(jT_s)$$

that

- Above eq. shows that is essentially an accumulated version of the quantizer output for the error signal $e(kT_s) - \hat{x}(kT_s)$. also gives a clue to the demodulator structure for DM. Fig. 10 shows a scheme for demodulation.

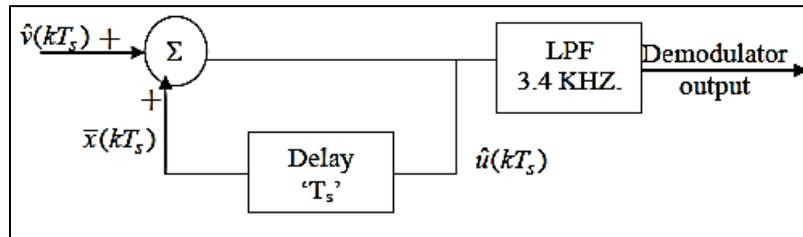


Fig.10 Demodulator Structure for DM

- The input to the demodulator is binary sequence and the demodulator normally starts with no prior information about the incoming sequence.
- Now, let us recollect from our discussion on DPCM in the previous lesson that, $u(kT_s)$ closely represents the input signal with small quantization error $q(kT_s)$, i.e.
 $u(kT_s) = x(kT_s) + e(kT_s)$
- Next, from the close loop including the delay-element in the accumulation unit in the Delta modulator structure, we can write

$$u([k-1]T_s) = \hat{x}(kT_s) = x(kT_s) - e(kT_s) = x([k-1]T_s) + q([k-1]T_s)$$

Hence, we may express the error signal as,

$$e(kT_s) = \{x(kT_s) - x([k-1]T_s)\} - q([k-1]T_s)$$

That is, the error signal is the difference of two consecutive samples at the input except the quantization error (when quantization error is small).

3.2 Advantages of a Delta Modulator Over DPCM

As one sample of $x(kT_s)$ is represented by only one bit after delta modulation, no elaborate word-level synchronization is necessary at the input of the demodulator. This reduces hardware complexity compared to a PCM or DPCM demodulator. Bit-timing synchronization is, however, necessary if the demodulator is implemented digitally. Overall complexity of a delta modulator-demodulator is less compared to DPCM as the predictor unit is absent in DM.

3.3 Limitations of DM: Slope Over Load Distortion

If the input signal amplitude changes fast, the step by step accumulation process may not catch up with the rate of change as shown in Fig 10.

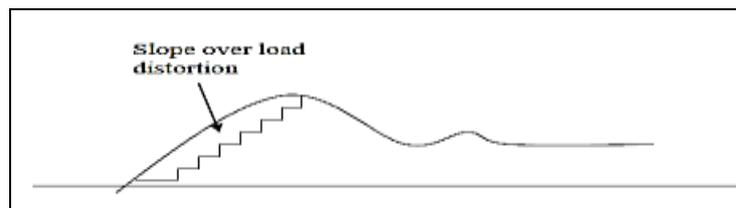


Fig 11 Slope-Overload Problem

- An intuitive remedy for this problem is to increase the step-size δ but that approach has another serious problem given below.

3.4 Granular Noise

If the step-size is made arbitrarily large to avoid slope-overload distortion, it may lead to 'granular noise'. Imagine that the input speech signal is fluctuating but very close to zero over limited time duration. This may happen due to pauses between sentences or else. During such moments, our delta modulator is likely to produce a fairly long sequence of 101010...., reflecting that the accumulator output is close but alternating around the input signal. This phenomenon is manifested at the output of the delta demodulator as a small but perceptible noisy background. This is known as 'granular noise'. A more efficient approach of adapting the step-size, leading to Adaptive Delta Modulation (ADM),

3.5 Condition for Avoiding Slope Overload

We may observe that if an input signal changes more than half of the step size (i.e. by 's') within a sampling interval, there will be slope-overload distortion. So, the desired limiting condition on the input signal $x(t)$ for avoiding slope-overloading is,

$$\left. \frac{dx(t)}{dt} \right|_{\max} \leq \frac{s}{T_s}$$

3.6 Comparison in PCM, DPCM & DM

Characteristics	PCM	DPCM	DM
Principle	Each discrete sample is quantized, encoded & sent.	Difference between consecutive samples is quantized, encoded & sent.	Sampling rate > Nyquist sampling rate so amplitude-to-sample amplitude difference is very low

			about 1-bit quantization which is encoded & send
Redundant Information	Carries redundant information.	Carries Less redundant information.	Carries high redundant information than PCM.
Bit rate generated	Higher compare to DPCM	Very Low compare to PCM	Higher than PCM
No. of Quantization levels.	High compare to DPCM , DM	Less compare to PCM	Less compare to DPCM, PCM
Quantization Noise	High compare to DPCM	Less compare to PCM, DM	High compared to PCM , DPCM due to step size called as Slope overload error & Granular Noise
Predictor Requirement	No	Yes	No, instead single Delay element is used.
Advantages	In PCM signal is regenerated so effects of amplitude, phase & nonlinear effects in one link has no effect on next link.	Less bit rate generated so better utilization of bandwidth.	Due to one bit quantization, no elaborate word-level synchronization is necessary at the input of the demodulator. This reduces hardware complexity compared to a PCM or DPCM demodulator.
	Transmission requirement PCM link are independent of total length of system.	Redundant information is less carried	Overall complexity of a delta modulator-demodulator is less compared to DPCM as the predictor unit is absent in DM.
Disadvantages	High bit rate & noise limits use	Predictor increase hardware complexity of system.	Higher Quantization noise compared to PCM ,DPCM
Application	Telephone Speech	Video Chatting on internet.	Video streaming

IV. CONCLUSION

Analog source encoding methods are divided into three types. Temporal waveform coding, Spectral waveform coding, Model-based coding. The minimum rate in bits per source output that is required to represent the output X of the memory less source with a distortion less than or equal to D is called the rate-distortion function $R(D)$. Note that $R(D)$ decreases as D increases. PCM is very rugged system, means less noise effect unless noise amplitude is greater than half of pulse height. In DPCM Less bit rate generated so better utilization of bandwidth. DM reduces hardware complexity compared to a PCM or DPCM demodulator.

REFERENCES

- [1] Digital Communication by Simon Hykin
- [2] NPTEL notes.
- [3] Digital Communication by John Proakis