International Journal of Electrical and Electronics Engineers
Vol. No. 9, Issue No. 01, January-June 2017
ISSN (O) 2321-2055
ISSN (P) 2321-2045

WAVELETS AND ITS APPLICATION IN IMAGE PROCESSING

Urvashi Bhat[1], Ruchika Singh[2]

[1,2]Deptt. of Electronics & Telecommunication Engineering. , GSMCOE, Balewadi, Pune (India)

ABSTRACT

In the past several years, wavelets and its applications have been one of the fastest-growing research areas. Wavelet theory has been employed in numerous fields and applications, such as signal and image processing, communication systems, radar, biomedical imaging, and air acoustics. It has emerged as a powerful tool in a number of engineering applications due to its ability for analyzing rapidly changing transient and non-stationary signals. It is especially valuable because of its ability to simultaneously provide local spectral and temporal information in a more flexible way. It is a method to produce a time–frequency decomposition which separates individual components more effectively than the traditional methods of signal analysis making it the first choice of researchers for analysis.

Keywords: Wavelet, Wavelet transform, Haar wavelet

I. INTRODUCTION

For the signal analysis, Fourier transform is the traditional mathematical technique used for transforming the signal from time domain to frequency domain .It breaks down signal into constituent sinusoids of different frequencies. The Fourier transform X(f) of the signal x(t) is given by

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-2\pi f t}\, dt \quad (1)$$

In transforming signal to the frequency domain, for the sake of perfect frequency resolution its temporal information is lost. Due to this reason, transient features within the signal are obscured.

This limitation was partly overcome by Short Time Fourier Transform (STFT) introduced by Dennis Gabor (1946) given by

$$ST\,F\,T(X, V)\,(\tau_0, \Omega_0) = \int_{-\infty}^{\infty} x(t)\,\{v(t - \tau_0)\, e^{j\Omega_0 t}\}^* \, dt \quad (2)$$

where the arguments in the first bracket are the secondary arguments namely X, V and the second bracket holds the primary arguments. A sliding time window of fixed length analyzes only a small section of the signal at a

time using a technique called windowing. It provides some information about both when and at what frequencies a signal event occurs. However, the information is obtained with limited precision determined by the size of the window.

In order to overcome this limitation, wavelet transform is introduced which is a windowing technique with variable-sized window. In Wavelet analysis, the window is shifted along the signal and for every position the spectrum is calculated. Then this process is repeated many times with a slightly shorter (or longer) window depending on the frequency. In the end the result will be a collection of time-frequency representations of the signal, all with different resolutions. Because of this collection of representations wavelet analysis is considered as multiresolution analysis technique. It allows the use of a shorter window for higher frequencies and a longer one for lower frequencies.

It is especially valuable because of its ability to elucidate simultaneously local spectral and temporal information from a signal in a more flexible way than the STFT by employing a window of variable width. In this way both short duration, high frequency and longer duration, lower frequency information can be captured simultaneously. Hence the method is particularly useful for the analysis of transients, aperiodicity and other non-stationary signal features.

Another key advantage of wavelet techniques is the variety of wavelet functions available, thus allowing the most appropriate to be chosen for the signal under investigation. This is in contrast to Fourier analysis which is restricted to only sinusoids [1].

II. WAVELETS

A wavelet is a wave-like oscillation of limited duration that has an average value of zero. Wavelets are functions that are used to decompose signals into many shifted and scaled representations of the original mother wavelet. Wavelets can be symmetric or asymmetric, sharp or smooth, regular or irregular.The most popular continuous wavelets are Mexican hat wavelet and the Morlet wavelet. These are described as follows.

2.1. The Mexican hat wavelet:

The Mexican hat wavelet is the second derivative of aGaussian function given by

$$\psi(t) = (1 - t^2)\, e^{\frac{-t}{2}} \quad (3)$$

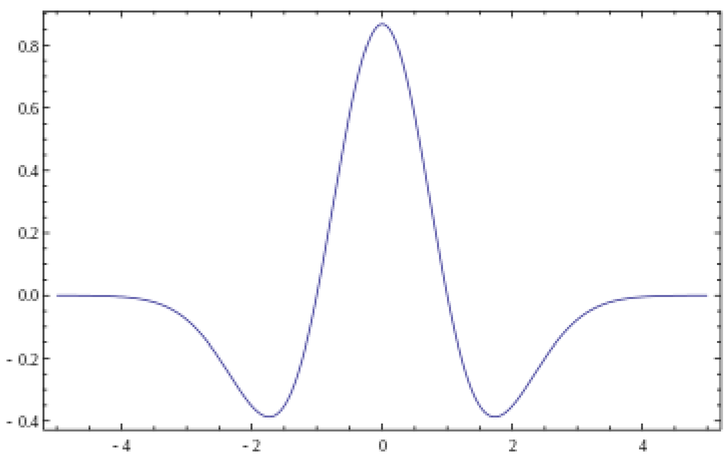

Fig.1 Mexican hat wavelet

2.2 The Morlet wavelet:

The Morlet wavelet is defined as:

International Journal of Electrical and Electronics Engineers
Vol. No. 9, Issue No. 01, January-June 2017
ISSN (O) 2321-2055
ISSN (P) 2321-2045

$$\psi(t) = \frac{1}{\sqrt[4]{\pi}}\left(e^{i\omega_0 t} - e^{-\frac{\omega_0^2}{2}}\right)e^{-\frac{t^2}{2}} \quad (4)$$

where ω_o is the central frequency of the mother wavelet

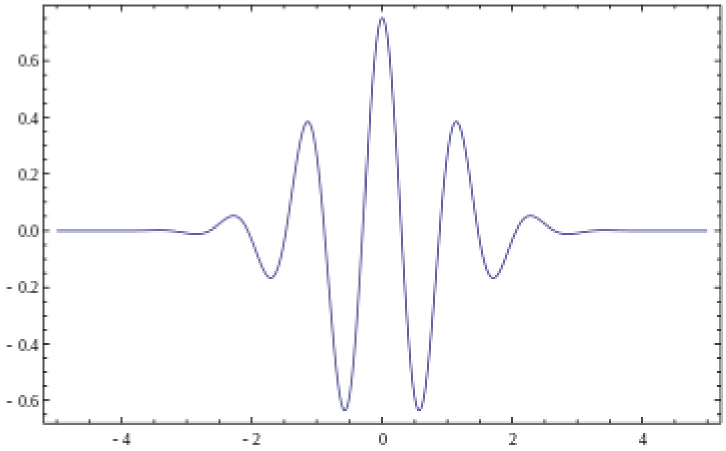

Fig.2. Morlet wavelet

III. WAVELET TRANSFORM:

The wavelet transform computes the inner products of a signal with a family of wavelets. The wavelet transform tools are categorized as:

1. Continuous Wavelet Transform
2. Discrete Wavelet Transform.

3.1. Continuous Wavelet Transform:

The continuous wavelet transform (CWT) is a time–frequency analysis method given by the equation

$$T(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} x(t)\,\psi^*\left(\frac{t-b}{a}\right) \quad (5)$$

Where $\psi^*(t)$ is the complex conjugate of the analyzing wavelet function ψ (t) commonly called as mother wavelet, a is the scale parameter of the wavelet and b is the translation parameter of the wavelet.

The **translation parameter** is related to the location of the window, as the window is shifted through the signal. This term corresponds to time information in the transform domain.

The **scale parameter**in the wavelet analysis is inversely related to frequency. Low frequencies (high scales) correspond to a global information of a signal, whereas high frequencies (low scales) correspond to a detailed information of a hidden pattern in the signal [2].

In order to be classified as a wavelet, a function must satisfy certain mathematical criteria. These are:

1) It must have finite energy:

$$E = \int_{-\infty}^{\infty} |\psi(t)|^2\, dt < \infty \quad (6)$$

2) If $\hat{\psi}(f)$ is the Fourier transform of $\psi(t)$, i.e.

$$\hat{\psi}(f) = \int_{-\infty}^{\infty} \psi(t)\, e^{-i2\pi f t}\, dt \quad (7)$$

International Journal of Electrical and Electronics Engineers
Vol. No. 9, Issue No. 01, January-June 2017
ISSN (O) 2321-2055
ISSN (P) 2321-2045

then the following condition must hold:

$$Cg = \int_0^\infty \frac{|\psi^{\wedge}(f)|^2}{f}\, df < \infty \qquad (8)$$

Equation (8) is known as the admissibilitycondition and Cg is called the admissibility constant. The value of Cg depends on thechosen wavelet.

3) For complex (or analytic) wavelets, the Fourier transform must both be real and vanish for negative frequencies [2].

3.2. Discrete wavelet Transform:

CWT provides highly redundant information which requires a significant amount of computation time and resources. In order to overcome this redundancy, discrete wavelet transform (DWT) was introduced which provides sufficient information for signal analysis and synthesis. DWT has the below given form

$$\Psi_{m,n}(t) = \frac{1}{\sqrt{a_0^m}}\,\psi\left(\frac{t - n b_0 a_0^m}{a_0^m}\right) \qquad (9)$$

where the integers *m* and *n* control the wavelet dilation and translation respectively; a_0 and b_0 is a fixed dilation step parameter.The most popular discrete wavelets are Haar wavelet and Daubchies wavelet [3]. These are described as follows.

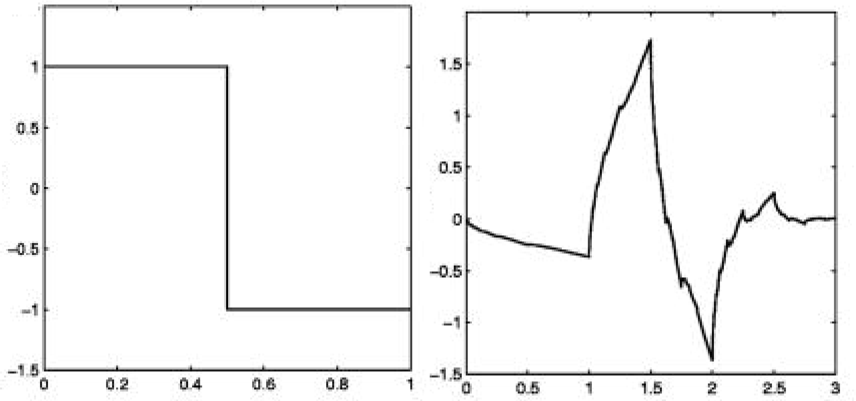

Fig.3. Haar Wavelet & Daubchies Wavelet

IV. APPLICATIONS OF WAVELETS IN IMAGE PROCESSING:

The applications of wavelets in image processing are often synonymous with compression and denoising. Wavelets provide efficient solutions in the domain of image compression.The objective for image compression is to save memory required to store the image, transmission band width and transmission time. Actually, image compression is the process of reducing the amount of data required to represent an image. As the image contains neighboring pixels which are correlated and therefore contain redundant information therefore image compression is achieved by removing the redundant information [4]. Basically, there are three types of redundancies:

a) Coding redundancy

b) Inter pixel redundancy

c) Psycho visual redundancy

V. RESULTS & DISCUSSION:

The Proposed methodology is to achieve high compression ratio in images using 2D Haar Wavelet using MATLAB software. The size of the original image and compressed image is 256 × 256 and 128 × 128.

.

Fig4. Original Image and Compressed image

5.1 IMPLEMENTATION:

a) **Decomposition:** Select Haar wavelet and level N. Compute the DWT. Decompose the signals at level N.

b) **Thresholding:** For each level from 1 to N, a threshold is selected and hard thresholding is applied to the detail coefficients.

c) **Reconstruction:** Compute wavelet reconstruction using the original approximation coefficients of level N using inverse DWT.

VI. CONCLUSION

The wavelet transform has emerged as an excellent time–frequency analysis and coding tool for image processing. Compression based on wavelet provides better image quality by reducing errors and give best results by exploiting various types of redundancies. It is expected to see an increased amount of research and technology development work in the coming years employing wavelets for various engineering applications.

VII. ACKNOWLEDGEMENT

We would like to acknowledge and extend heartiest gratitude to our Head of the Department**, Mr. S. S. Khonde**, for his motivation, encouragement and support. This work would not have been possible without the encouragement of our family and friends.

REFERENCES

[1] Paul S Addison, "Wavelet transforms and the ECG: a review", Journal on Physiological Measurement, pp. 155-199, 2005

[2] Robi Polekar, "The Wavelet Tutorial"

[3] Daniel T.L.Lee, Akio Yamamoto, "Wavelet analysis: Theory & Applications", Hewlett Packard Journal, pp. 147-179.

[4] Bhonde Nilesh, Shinde Sachin, Nagmode Pradip, D.B. Rane, "Image Compression Using Discrete Wavelet Transform" International Journal of Computer Technology and Electronics Engineering (IJCTEE) Volume 3, Special Issue, March-April 2013.

[5] S. Bhavani, K. Thanushkodi, "A Survey on Coding Algorithms in Medical Image Compression", International Journal on Computer Science and Engineering, Vol. 02, No. 05, pp. 1429-1434, 2010.

[6] G. K. Kharate, V. H. Pati, "Color Image Compression Based On Wavelet Packet Best Tree", International Journal of Computer Science, Vol. 7, No.3, March 2010.

[7] M. R. Haque and F. Ahmed, "Image data compression with JPEG and JPEG2000", 8th International Confrrence on Computer and Information Technology, pp. 1064-1069, 2005.

[9] A. Alice Blessie1, J. Nalini and S.C.Ramesh, "Image Compression Using Wavelet Transform Based on the Lifting Scheme and its Implementation", IJCSI International Journal of Computer Science Issues, Vol. 8, Issue 3, No. 1, May 2011.

[8] S. Bhavani, K. Thanushkodi, "A Survey on Coding Algorithms in Medical Image Compression", International Journal on Computer Science and Engineering, Vol. 02, No. 05, pp. 1429-1434, 2010.

[9] G. K. Kharate, V. H. Pati, "Color Image Compression Based On Wavelet Packet Best Tree", International Journal of Computer Science, Vol. 7, No.3, March 2010.

[10] M. R. Haque and F. Ahmed, "Image data compression with JPEG and JPEG2000", 8th International Confrrence on Computer and Information Technology, pp. 1064-1069, 2005.