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Performance Analysis of MIMO System using Space Division Multiplexing Algorithms

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ABSTRACT

The bit error rate (BER) of Multiple Input Multiple Output (MIMO) system using Binary Phase Shift Modulation (BPSK) on Rayleigh fading channels is analyzed. Minimum Mean Squared Error, Zero Forcing and Zero Forcing with Successive Interference Cancellation algorithms are used. These Space Division Multiplexing (SDM) algorithms are programmed in MATLAB and some simulations are performed to obtain BER characteristics. These characteristics are used to compare the performance of the different SDM algorithms. In all simulations it is assumed that the channel is perfectly known to the receiver.

Keywords-*dipole configurations; MIMO;Bit Error Rate (BER) ,Space Division Multiplexing(SDM)*

I. INTRODUCTION

In the last few years wireless services have become more and more important. Likewise the demand for higher network capacity and performance has also been increased. Several options like higher bandwidth, optimized modulation or even code multiplex systems offer practically limited potential to increase the spectral efficiency. In MIMO, both transmitter and receiver are provided with more than one antenna. MIMO performs well in scattering rich environment. The channel capacity increases linearly with number of antennas if multiple antennas are used at both ends [1]. For rich scattering environment channel it is possible to increase the data rate by transmitting separate information streams on each antenna. For example, using four transmit and four receive antennas, four times the capacity of a single antenna system can be achieved [2].

For coherent communication systems, error performance are usually evaluated by assuming that a perfect phase reference is available in the receiver for demodulation [3,4]. In practice, this local phase reference is however reconstructed from a noise-corrupted version of a received signal, and thus a phase error, θ, is usually resulted. The immediate effect of the phase error is degradation of detection performance of the coherent systems. Over the years, many researchers have investigated the error performance of binary phase shift keying (BPSK) and differential PSK (DPSK) systems over an additive white Gaussian noise (AWGN) channel in the presence of noisy phase reference [5,6]. Here we evaluate the bit error rate (BER) of the BPSK systems in the presence of Rayleigh fading and noisy phase reference. Throughput can be increased by simultaneously transmitting different streams of data on the different transmit antennas but at the same carrier frequency. Although these parallel data streams are mixed up in the air, they can be recovered at the receiver by using spatial sampling (i.e multiple receive antennas) and corresponding signal processing algorithms, provided that the MIMO channel is well conditioned. Quality of service is improved through space diversity by transmitting same signal over multiple antennas [7]. Three approaches to diversity are frequency diversity, time diversity and space diversity. In frequency diversity, the information bearing signal is transmitted by means of several carriers that are spaced sufficiently apart from each other to provide independently fading versions of the signal. In time diversity, the same information bearing signal is transmitted in different time slots, with the interval between successive time slots being equal to or greater than the coherence time of the channel. In space diversity, multiple transmit or receive antennas, or both are used. space diversity on receive, using four techniques for its implementation, namely selection combining, maximal ratio combining, equal gain combining and square law combining describes a mathematical model of MIMO wireless communications.

The organization of the paper is as follows. Section II briefly reviews the Rayleigh fading channel model. Section III gives the BER analysis of SDM algorithm. Section IV discusses the results.

II. RAYLEIGH FADING CHANNEL

When no strong LOS or specular path is present, the large number of reflectors within a typical indoor-like environment results in Rayleigh fading. For a MIMO system operating in such a rich-scattering environment, when the antenna spacing is chosen equal to or larger than half the carrier wavelength, the channel coefficients can be assumed independent identically distributed (i.i.d). The complex envelope of the received signal at the antenna array after matched filtering is given by

$$y = Hx + n \qquad \text{---(1)}$$

where x is the transmit vector, y is the receive vector, H is the $N_R \times N_T$ channel matrix, and n is the additive white Gaussian noise (AWGN) vector at a given instant in time. Throughout the paper, it is assumed that the channel matrix is random and that the receiver has perfect channel knowledge [1]. It is also assumed that the channel is memoryless, i.e., for each use of the channel an independent realization of H is drawn. A general entry of the channel matrix is denoted by $\{h_{ij}\}$. This represents the complex gain of the channel between the j[th] transmitter and the i[th] receiver. With a MIMO system consisting of N_T transmit antennas and N_R receive antennas, the channel matrix is written as

$$H = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & .. & h_{2n} \\ .. & . & . & . \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{pmatrix} \qquad \text{---(2)}$$

In a rich scattering environment with no line of sight (LOS), the elements of the dimensional channel transfer matrix are i.i.d.circularly-symmetric complex Gaussian variables with zero mean and unit variance, with an independent realization. The definition of a circularly-symmetric complex Gaussian random variable, say z, with zero mean and variance σ^2 is given by $z = x + iy$ with x and y being i.i.d. zero mean real Gaussian variables with variance $\sigma^2/2$.

The probability density function of h is given by,

$$p(h) = \frac{h}{\sigma^2} e^{-h^2/2\sigma^2}, z > 0 \qquad \text{---(3)}$$

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This model is called Rayleigh fading channel model and this is reasonable for an environment where there are large numbers of reflectors.

III. SPACE DIVISION MULTIPLEXING ALGORITHMS

If the wireless communication channel is richly scattered, a distinction can be made depending on to what extent the algorithms exploit the transmit diversity provided by the channel. On the one hand, transmit diversity schemes fully use the spatial dimension for adding more redundancy, thus keeping the data rate equivalent to a single antenna system. Spatial multiplexing algorithms exploit the spatial dimension by transmitting multiple data streams in parallel on different antennas, to achieve high data rates. These algorithms are referred to as Space Division Multiplexing (SDM) algorithms [8]. The main advantages of SDM are that it directly exploits the MIMO channel capacity to improve the data rate. Zero Forcing (ZF), Minimum Mean Squared Error (MMSE) and Zero Forcing with Successive Interference Cancellation are Space Division Multiplexing algorithms [9].

A. Zero Forcing (ZF)

Zero forcing SDM algorithms is a linear MIMO technique, the processing takes place at the receiver where, under the assumption that the channel transfer matrix H is invertible, H is inverted and the transmitted MIMO vector s is estimated by

$$s_{est} = H^{-1}x \qquad \text{---(4)}$$

In this technique each substream in turn is considered to be the desired signal, and the remaining data streams are considered as interferers. Nulling of the interferers is performed by linearly weighting the received signals such that all interfering terms are cancelled. For zero forcing, nulling of the interferers can be performed by choosing 1*Nr dimensional weight vector w^i (with i=1,2,…,Nt) referred to as nulling vectors[9], such that

$$w^i h_p = \begin{cases} 0, & p \neq i \\ 1, & p = i \end{cases} \qquad \text{---(5)}$$

where h_p denotes the p-th column of the channel matrix H. Let w^i be the i[th] row of a matrix W, then it follows that $WH = I_{N_t}$, where W is a matrix that represents the linear processing in the receiver. So, by forcing the interferers to zero, each desired element of s can be estimated. If H is not square, W equals the pseudo-inverse of H

$$W = H^{+} = (H^H H)^{-1} H^H \qquad \text{---(6)}$$

B. Minimum Mean Squared Error (MMSE)

The minimum mean square error approach tries to estimate a random vector s on the basis of observations x is to choose a function f(x) that minimizes the mean square error (MSE), an exact function f(x) is usually hard to obtain, however if we restrict this function to be a linear function of the observations, an exact solution can be achieved.

$$\varepsilon^2 = E\left[(s - s_{est})^H (s - s_{est})\right] = E\left[(s - f(x))^H (s - f(x))\right] \text{---(7)}$$

Using linear processing, the estimates of s can be found by

$$s_{est} = Wx \qquad \text{---(8)}$$

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Now, to obtain the linear minimum mean square error solution, W must be chosen such that the mean square error ε^2 is minimized:

$$\varepsilon^2 = E\left[(s - s_{est})^H (s - s_{est})\right] = E\left[(s - Wx)^H (s - Wx)\right] \qquad \text{---(9)}$$

C. Zero Forcing with Successive Interference Cancellation (ZF-SIC)

The linear approaches are viable, but the superior performance is obtained if non-linear techniques are used. In successive interference cancellation (SIC) first the most reliable element of the transmitted vector s could be decoded and used to improve the decoding of the other elements of s, a better performance can be achieved and it exploits the timing synchronism inherent in the system model. Furthermore linear nulling (ZF or MMSE) is used to perform the detection. In other words, SIC is based on the subtraction of interference of already detected elements of s from the receiver signal vector x. this result in a modified receiver vector in which effectively fewer interferers are present. When SIC is applied, the order in which the components of s are detected is important to the overall performance of the system. To determine a good detection order, the covariance matrix of the estimation error is used. The covariance matrix is given by

$$Q = E\left[(s - s_{est})(s - s_{est})^H\right] = \sigma_n^2 (H^H H)^{-1} \qquad \text{---(10)}$$

The decoding algorithm consists of three parts:

1. Ordering: determine the transmitted stream with the lower error variance.
2. Interference Nulling: estimate the strongest transmitted signal by nulling out all the weaker transmitted signals.
3. Interference cancellation: remodulate the data bits, subtract their contribution from the received signal vector and return to the ordering step.

More detailed description of the above three recursive steps is

1. Compute H^+ find the minimum squared length row of H^+ say it is the p-th, and permute it to be the last row. Permute the columns of H accordingly.
2. from the estimate of the corresponding element of s, in case of ZF:

   $$(S_{est})_p = w^{Nt} x \qquad \text{---(11)}$$

   where the weight vector w^{Nt} equals row N_t of the permuted H^+. Slice $(S_{est})_p$ to the nearest constellation point $(S_{est,sliced})_p$

3. while Nt-1>0 go back to step 1, but now with:

   $H \rightarrow H^{(Nt-1)} = (h_1, h_2, \ldots h_{Nt-1}), x \rightarrow h_{Nt}(S_{est,sliced})_p$ and $N_t \rightarrow N_t - 1$.

further simplification is possible when the QR decomposition is used. Assume that the recursive process is in its (k+1) the run, then the dimensions of H are determined with the original Nt. Based on the QR decomposition, we may write $H = Q_{QR} R$ then the weight vector becomes

$$w^{Nt-k} = \frac{1}{r_{(Nt-k)(Nt-k)}} q_{Nt-k}^H \qquad \text{---(12)}$$

where r_{yy} denotes element (y,y) of R and q_y the y-th column of Q_{QR}. with respect to ZF, the ZF with SIC algorithm introduces extra complexity in the preamble phase as well as in the payload phase.

IV. BER ANALYSIS USING SDM ALGORITHMS

The average BER for the BPSK system in the presence of Rayleigh fading and noisy phase reference is considered in this section. For fading channels, the conditional BER for the BPSK with phase error is given by [4]

$$P_e(\gamma,\theta)=\frac{1}{2}erfc(\sqrt{\gamma}\cos\theta) \qquad \text{---((13)}$$

where *erfc* (.) is the complementary error function and γ is the instantaneous signal to noise ratio (SNR) per bit of the received signal. The phase error θ is assumed to be uniformly distributed in a range of $(-\phi,\phi)$ and the probability density function (pdf) of it is given by

$$p(\theta)=1/2\phi\ . \qquad \text{---(14)}$$

In addition, the pdf of γ for the Rayleigh fading channel is given by

$$p(n)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(n-\mu)^2}{2\sigma^2}} \qquad \text{---(15)}$$

with $\mu=0 and \sigma^2=\frac{N_0}{2}$.

For BPSK modulation in Rayleigh fading channel, the bit error rate is derived as,

$$P_b=\frac{1}{2}\left(1-\sqrt{\frac{(E_b/N_0)}{(E_b/N_0)+1}}\right) \qquad \text{---(16)}$$

A. SDM Algorithm Description

a) Generate the random binary sequence of +1's and -1's.

b) Group them into pair of two symbols and send two symbols in one time slot

c) Multiply the symbols with the channel and then add white Gaussian noise.

d) Equalize the received symbols.

e) Perform hard decision decoding and count the bit errors. In Zero Forcing Equalizer with Successive Interference Cancellation (ZF-SIC) approach, after equalization take the symbol from the second spatial dimension, subtract from the received symbol and then perform Maximal Ratio Combining for equalizing the new received symbol.

V. RESULTS AND DISCUSSION

Fig.1 shows Eb/No in dB versus Bit Error Rate (BER), the probability of bit-error for QPSK is the same as for BPSK: However, in order to achieve the same bit-error probability as BPSK, QPSK uses twice the power (since two bits are transmitted simultaneously).

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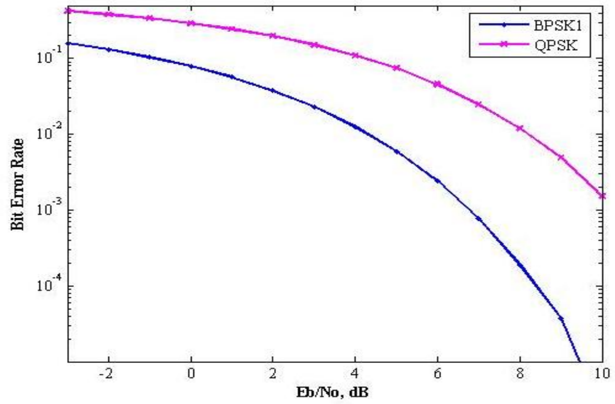

Fig.1. Eb/No in dB versus Bit Error Rate (BER), for QPSK and BPSK

Fig.2 shows BER for 2*2 MIMO channel with zero forcing equalizer in Rayleigh channel, the off diagonal terms in the matrix $H^H H$ are not zero. Because the off diagonal terms are not zero, the zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e when solving for x_1 the interference from x_2 is tried to be nulled and vice versa. While doing so, there can be amplification of noise. Hence Zero Forcing equalizer is not the best possible equalizer to do the job. However, it is simple and reasonably easy to implement. Further, it can be seen that, following zero forcing equalization, the channel for symbol transmitted from each spatial dimension (space is antenna) is a like a 1×1 Rayleigh fading channel. Hence the BER for 2×2 MIMO channel in Rayleigh fading with Zero Forcing equalization is same as the BER derived for a 1×1 channel in Rayleigh fading. The Zero Forcing equalizer is not the best possible way to equalize the received symbol. The zero forcing equalizer helps us to achieve the data rate gain, but not take advantage of diversity gain (as we have two receive antennas).

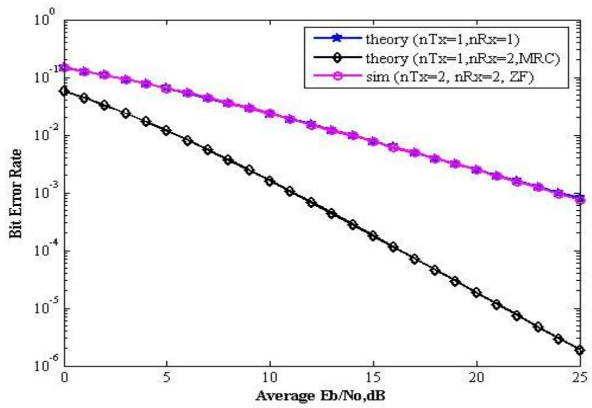

Fig.2. BER plot for 2×2 MIMO channel with ZF equalizer (BPSK modulation in Rayleigh channel)

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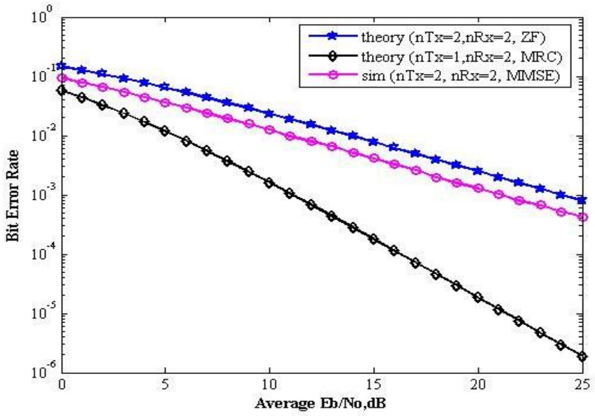

Fig.3. BER plot for 2×2 MIMO with MMSE equalization for BPSK in Rayleigh channel

Fig.3 shows the BER in a 2*2 MIMO channel with MMSE equalization. BER of BPSK modulation in 2*2 MIMO with zero forcing-Successive Interference Cancellation equalization shown in fig.4. Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply Maximal Ratio Combining (MRC).MMSE and ZF-SIC reduces the bit error rate compare to zero forcing algorithm.

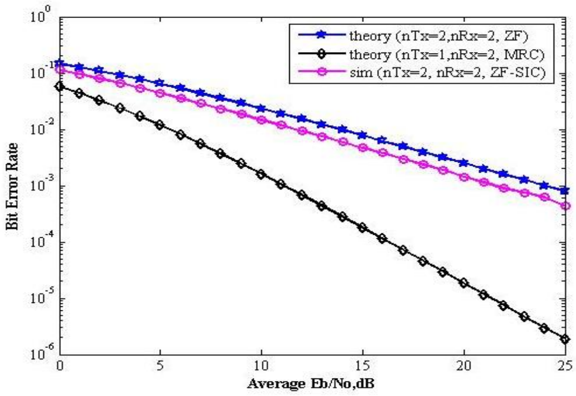

Fig.4.BER plot for BPSK in 2×2 MIMO channel with Zero Forcing Successive Interference Cancellation equalization

VI. CONCLUSION

In this paper the Bit Error Rate (BER) of BPSK modulation is analyzed in Rayleigh fading channel model with zero forcing equalization techniques. This result is compared with Minimum Mean Square Error (MMSE) and zero forcing with successive interference cancellation techniques. Zero Forcing equalizer is not the best possible equalizer but it is simple and reasonably easy to implement.

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